Core 2 Jan 2008 $\boxed{72}$

1) Area sector $= \frac{1}{2}r^2\theta = \frac{1}{2}\times 11^2 \times 0.7$

$= 42.35$ (2)

Area $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}11^2 \times \sin 0.7$

$\boxed{4}$ $= 38.98$ (1)

Area segment $= 42.35 - 38.48$

$= 3.37$ (1)

2) width strip $= \frac{7-1}{3} = 2$ (1)

$I = \frac{2}{2}\left(2 + 2(\sqrt{12}+\sqrt{28}) + \sqrt{52}\right)$ (2)

$\boxed{4}$ $= 26.7$ (1)

ie $\frac{2}{2}\left(2+2(3.46+5.29)+7.211\right)$

3) i) $\log_a 2 + \log_a 3 = \log_a 6$ (1)

ii) $2\log_{10} x - 3\log_{10} y = \log_{10} x^2 - \log_{10} y^3$ (1)

$= \log\left(\frac{x^2}{y^3}\right)$ (2)

$\boxed{4}$

4) i) $\frac{BD}{\sin 62} = \frac{16}{\sin 50}$ (1) $BD = \frac{16\sin 62}{\sin 50}$

$= 18.4\text{ cm}$ (1)

ii) $\cos\theta = \frac{10^2+20^2-18.4^2}{2\times 10\times 20}$ (1) $= 0.3998$ (1)

$\boxed{5}$ $\theta = 66.4°$ (1)

5) $y = \int 12x^{\frac{1}{2}}\,dx = 8x^{\frac{3}{2}} + c$ (3)

subst $x = 4$, $y = 50$

$50 = 8\times 8 + c \quad c = -14$ (2)

$\boxed{6}$ $y = 8x^{\frac{3}{2}} - 14$ (1)

6) i) $n=1 \quad u_1 = 7$ so

$n=2 \quad u_2 = 9$

$\boxed{8}$ $n=3 \quad u_3 = 11$ (2)

ii) AP $a = 7$ + $d = 2$ (1)

iii) $S_n = 2200 = \frac{N}{2}\left(2\times 7 + (N-1)\times 2\right)$ (2)

$4400 = 12N + 2N^2$

$N^2 + 6N - 2200 = 0$ (1)

6 iii) $(N-44)(N+50) = 0$

$N = 44$ (1)

7) i) Area below is $-$ve

Area above is $+$ve (1)

ii) $A = \int_0^3 x^2 - 3x\,dx + \int_3^5 x^2 - \frac{3x}{2}\,dx$ (2)

$\boxed{8}$ $A_1 = \left[\frac{x^3}{3} - \frac{3x^2}{2}\right] = \left(9 - \frac{27}{2}\right) - 0 = -4\frac{1}{2}$ (2)

$A_2 = \left[\frac{x^3}{3} - \frac{3x^2}{2}\right] = \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right)$

$= 8\frac{2}{3}$ (2)

Total $= 8\frac{2}{3} + 4\frac{1}{2}$

$= 13\frac{1}{6}$ (1)

8) i) $a = 10 \quad r = 0.8$ (2)

$U_4 = ar^3 = 10\times 0.8^3 = 5.12$

ii) $S_{20} = a\frac{(1-r^{20})}{1-r} = \frac{10(1-0.8^{20})}{1-0.8}$

$= 49.4$ (2)

iii) $S_\infty = \frac{a}{1-r} = \frac{10}{1-0.8} = 50$ (1)

$S_\infty - S_n = 50 - \frac{10(1-0.8^N)}{0.2}$

$\boxed{11}$ $= 50 - 50(1-0.8^N)$ (1)

but $S_\infty - S_N < 0.01$

so $50 - 50(1-0.8^N) < 0.01$ (1)

$\div 50 \quad 1 - (1-0.8^N) < 0.0002$

$0.8^N < 0.0002$ (1)

$\log 0.8^N < \log 0.0002$ (1)

$N\log 0.8 < \log 0.0002$ (1)

$N > 38.169$

ie $N = 39$ (1)

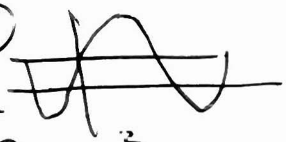
9 i) max $(90, 2)$ ↑ stretch × 2

min $(-90, -2)$ (2)

ii) a) 2nd soln $x = 180 - \alpha$ (1)

b) soln $x = -\alpha$ (1)

or $x = -180 + \alpha$



c) $2\sin x = 2 - 3\cos^2 x$

$2\sin x = 2 - 3(1 - \sin^2 x)$ (1)

$3\sin^2 x - 2\sin x - 1 = 0$ (1)

$(3\sin x + 1)(\sin x - 1) = 0$ (1)

$\sin x = -\frac{1}{3}$ or $+1$ (1)

$\boxed{10}$ ↑

$x = -19.5$ or $-180 + 19.5$

$= -19.5$ and -160.5 (2)

or $x = 90$ from 2nd bracket

10) i) $(2x+5)^4 = (2x)^4 + 4C1(2x)^3(5)^1 + 4C2(2x)^2(5)^2 + 4C3(2x)5^3 + 5^4$ (2)

$= 16x^4 + 160x^3 + 600x^2 + 1000x + 625$ (2)

ii) $(2x-5)^4 = 16x^4 - 160x^3 + 600x^2 - 1000x + 625$

ie replace 5 by -5 (2)

$(2x+5)^4 - (2x-5)^4 = 320x^3 + 2000x$ as x^4, x^2 + no.s disappear

$k = 2000$

iii) LHS $x = 2$ $(2x+5)^4 - (2x-5)^4 = 9^4 - (-1)^4 = 6560$

RHS $3680x - 800 = 7360 - 800 = 6560$

so $x = 2$ is a root (1)

but LHS $= 320x^3 + 2000x = 3680x - 800$

$\boxed{12}$ so $320x^3 - 1680x + 800 = 0$ (1)

$4x^3 - 21x + 10 = 0$ (1)

$x = 2$ is a factor do long div to get

$(x-2)(4x^2 + 8x - 5) = 0$ (1)

$(x-2)(2x-1)(2x+5) = 0$ (1)

$x = 2, \frac{1}{2}, -2\frac{1}{2}$ (1)